Convergence to Nash equilibria in ratio-bounded games

Francesco Caruso*, Maria Carmela Ceparano, Jacqueline Morgan

Abstract

In a two-player non-cooperative game framework, we deal with the affine relaxations of the so-called *best response algorithm*, that is an iterative scheme where the updated strategy of player i is obtained by selecting a minimizer of player i's payoff function given the strategy of the other player coming from the previous step. In order to be able to specify the convergence of any type of affine relaxation of the best response algorithm, we define a new class of convex games, called *ratio-bounded games*. This class contains games broadly used in literature (such as weighted potential and zero-sum games), both in finite and in infinite dimensional setting. Its definition relies on a unifying property and on three associate key-parameters explicitly related to the data. Depending on how the parameters are ordered, we provide a classification of the ratio-bounded games in four subclasses such that, for each of them, the following issues are answered when the strategy sets are real Hilbert spaces: existence and uniqueness of the Nash equilibria, global convergence of the affine relaxations of the best response algorithm, estimation of the related errors, determination of the algorithm with the highest speed of convergence and comparison with the known results.

References

[1] CHERRUAULT Y., LORIDAN P., Méthodes pour la recherche de points de selle, *J. Math. Anal. Appl.* **42** (3): 522 – 535, 1973.

[2] MORGAN J., Méthode directe de recherche du point de selle d'une fonctionnelle convexe-concave et application aux problèmes variationnels elliptiques avec deux contrôles antagonistes, *Int. J. Comput. Math.* **4** (1-4): 143 – 175, 1974.

*speaker; University of Naples Federico II, Italy, francesco.caruso@unina.it

[3] BAŞAR T., Relaxation techniques and asynchronous algorithms for on-line computation of non-cooperative equilibria, *J. Econom. Dynam. Control* **11** (4): 531 – 549, 1987.

[4] LI S., BAŞAR T., Distributed algorithms for the computation of noncooperative equilibria, *Automatica* **23** (4): 523 – 533, 1987.

[5] CARUSO F., CEPARANO M.C., MORGAN J., An inverse-adjusted best response algorithm for Nash equilibria, *SIAM J. Optim.* **30** (2): 1638 – 1663, 2020.