An augmented subgradient algorithm for minimizing nonsmooth DC functions

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Abstract

In this talk we consider an unconstrained DC optimization problem

$$\begin{cases} \text{minimize} & f(x) \\ \text{subject to} & x \in \mathbb{R}^n, \end{cases} \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $f(x) = f_1(x) - f_2(x)$ and functions $f_1, f_2 : \mathbb{R}^n \to \mathbb{R}$ are convex and in general, nonsmooth. Some important practical problems can be formulated as a nonsmooth DC program of the form (1). They include supervised data classification, cluster analysis, clusterwise linear regression analysis and edge detection problems; for more information see [1] and references therein.

To date, several methods have been developed to locally solve Problem (1). They include, but not limited to the aggregate subgradient method [2], the proximal bundle method with the concave affine model [4], the proximal bundle method for DC optimization [3] and the double bundle method [5].

We propose an augmented subgradient algorithm to solve problem (1) [1]. At each iteration, by using several subgradients of the first DC component and one subgradient of the second DC component of the objective function, a search direction is calculated. If the direction is descent, then an Armijo-type line search procedure to find the next iteration point is applied. Otherwise (the direction is not descent) by using new information of the both components of the objective function, a new search direction is computed and the algorithm is repeated. The global convergence is proved in the sense that, the sequence of points generated by the method converges to a critical point of the unconstrained DC optimization problem. At the end, in order to study the reliability and efficiency of the proposed algorithm, Fortran implementation of it is prepared and numerical experiments have been made by using some academic test problems with nonsmooth DC objective functions. In addition, the performance of the algorithm is compared with that of two general nonsmooth optimization solvers and five solvers specifically designed for unconstrained DC optimization. Computational results show that the developed method is efficient and robust for solving nonsmooth DC optimization problems.

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