Ohlin-type result for strongly convex functions and set-valued maps

Kazimierz Nikodem *

Abstract

Let $I \subset \mathbb{R}$ be an interval. A function $f : I \to \mathbb{R}$ is called *strongly convex with modulus* $c > 0$ if

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y) - ct(1-t)(x-y)^2$$

for all $x, y \in I$ and $t \in (0,1)$. Let $(\Omega, \mathcal{A}, P)$ be a probability space. Given a random variable $X : \Omega \to \mathbb{R}$ we denote by F_X, $\mathbb{E}[X]$ and $\mathbb{D}^2[X]$ the distribution function, the expectation and the variance of X, respectively. The following Ohlin-type result for strongly convex functions is presented:

Theorem. *Let $X, Y : \Omega \to I$ be square integrable random variables such that $\mathbb{E}[X] = \mathbb{E}[Y]$. If there exists $t_0 \in \mathbb{R}$ such that*

$$F_X(t) \le F_Y(t) \;\; if \;\; t < t_0 \quad and \quad F_X(t) \ge F_Y(t) \;\; if \;\; t > t_0,$$

then

$$\mathbb{E}[f(X)] - c\mathbb{D}^2[X] \le \mathbb{E}[f(Y)] - c\mathbb{D}^2[Y]$$

for every function $f : I \to \mathbb{R}$ strongly convex with modulus c.

As an application various inequalities related to strongly convex functions are obtained in a simple unified way. Finally, counterparts of the Ohlin theorem for convex and strongly convex set-valued maps are given.

References

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*University of Bielsko-Biala, Poland, knikodem@ath.bielsko.pl