Relationships between constrained and unconstrained multi-objective optimization and application in location theory

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Abstract

The main purpose of the lecture is to investigate relationships between constrained and unconstrained multi-objective optimization problems, see [1, 2]. We mainly focus on generalized convex multi-objective optimization problems, i.e., the objective function is a componentwise generalized convex (e.g., quasi-convex, semi-strictly quasi-convex or explicitly quasi-convex) function and the feasible domain is a not necessarily convex set. Beside the field of location theory, the assumptions of generalized convexity are found in several branches of Economics, e.g., in the field of utility theory.

In the talk, we formulate the basic constrained multi-objective optimization problem and the corresponding extended unconstrained one, we introduce solution concepts and recall results about generalized convex and semi-continuous functions.

Under suitable assumptions (e.g., generalized convexity assumptions), we derive a characterization of the set of (weakly) efficient solutions of a constrained multi-objective optimization problem using characterizations of the set of (weakly) efficient points of unconstrained multi-objective optimization problems. Furthermore, we present an assertion that provides lower and upper bounds for the sets of (weakly) efficient solutions for multi-objective optimization problems involving nonconvex constraints.

Moreover, we deduce necessary optimality conditions using our results concerning relationships between constrained and unconstrained multi-objective optimization problems. We apply these optimality conditions for deriving a proximal-point-algorithm for solving constrained multi-objective location problems, see [3].

Our results can be applied to constrained point-objective location problems involving mixed gauges defined by

$$\begin{pmatrix} \eta_1(x - a^1) \\ \dots \\ \eta_m(x - a^m) \end{pmatrix} \to \min_{x \in X},$$

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where $\eta_1, \ldots, \eta_m : \mathbb{R}^n \to \mathbb{R}$ represent special distance functions (gauges) and $a^1, \ldots, a^m$ are finitely many given points in $\mathbb{R}^n$. The MATLAB-based software tool

Facility **L**ocation **O**ptimizer (**FLO**)

can be used for solving special types of single- as well as multi-objective location problems involving different distances measures. For more information, see http://www.project-flo.de.

References

[1] GÜNTHER C. & TAMMER CHR., On generalized-convex constrained multi-objective optimization, *Pure and Applied Functional Analysis* **3** (3): 429 – 461, 2018.

[2] GÜNTHER C., TAMMER CHR. & YAO J.-C., Necessary optimality conditions in generalized convex multi-objective optimization involving nonconvex constraints, *Applied Analysis Optimization* **2** (3): 403–421, 2018.

[3] HEIN S., Lösung von restringierten mehrkriteriellen Optimierungsproblemen mittels Penalisierung und Proximal-Point-Algorithmus, *Master Thesis, University Halle*, 2022.